課題名（タイトル）：

Band structure of ultracold superfluid Fermi gases in an optical lattice

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1. 本課題の研究の背景、目的、関係するプロジェクトとの関係

Ultracold gases in optical lattices provide a new frontier of research where many remarkable phenomena can be observed and investigated. By using Feshbach resonances one can tune the interaction between atoms and investigate the crossover between the Bardeen-Cooper-Schrieffer (BCS) state and a Bose-Einstein condensate (BEC), passing through a resonant regime where the scattering length is very large and the system exhibits universal properties (unitary regime). Here, "universal" means that the properties of the system does not depend on details of interparticle potential. Therefore, study of unitary Fermi gases is also useful for other systems, such as neutron gases in inner crusts of neutron stars, where the neutron-neutron scattering length is much larger than the interparticle separation. Our final aim thus includes learning neutron star crusts by simulating neutron star matter using cold atom gases as well as understanding the cold atom gas systems themselves.

Interplay between the non-linearity and the periodic potential is one of the important issues of the ultracold atomic gases in optical lattices and it gives rise to various intriguing phenomena. However, unlike for BECs, little has been explored so far on this issue for superfluid Fermi gases in optical lattices. One of a handful exceptions is the swallowtail band structure in the BCS-BEC crossover which has been explored by ourselves in our previous project using RICC [G. Watanabe, S. Yoon, and F. Dalfovo, Phys. Rev. Lett. **107**, 270404 (2011)].

In 2012, in collaboration with Dr. Nakatsukasa, we started working on period doubling phenomenon in superfluid Fermi gases in optical lattices. Density wave is one of the most fundamental phenomena in condensed matter physics. In Bose-Einstein condensates (BECs) in a periodic potential, non-linearity of the interaction term gives rise to stationary states whose period does not coincide with that of the external potential; instead, a multiple of it. However, it has been found that the period-doubled states in BECs are energetically unfavorable compared to the normal Bloch states and in many cases they are also dynamically unstable.

As for superfluid Fermi gases, on the other hand, there is no study about period doubling and the problem is totally open. Superfluid Fermi gases in the BCS-BEC crossover provide a very intriguing question of how period-doubled states evolve between the BCS and BEC limits. The problem of the period doubling can be even more important in Fermi superfluids, due to the possible implications in superconducting electrons in solids and superfluid neutrons in neutron stars.

In this project, we study states with various periods (mainly, period 1, 2, and 3) of Fermi superfluids in a 1D periodic potential in the BCS-BEC crossover. The first purpose of this project is to explore whether we have period-doubled states in Fermi superfluids. If we find the existence of the period-doubled states in Fermi superfluids, we shall look into how they evolve along the BCS-BEC crossover and solve the following questions.

1) Difference and similarity between Fermi

superfluids and BECs. We shall pursue unique properties of period-doubled states of Fermi superfluids which are absent for BECs.

2) Stability of the period-doubled states along the BCS-BEC crossover. The main question is whether we have dynamically stable period-doubled states in some parameter region.

3) Experimental observability and consequences of period-doubled states in the BCS-BEC crossover.

4) A criterion for appearance of the period-doubled states in the BCS side.

2. 具体的な利用内容、計算方法

To study the period-doubling phenomena in ultracold superfluid Fermi gases, we use a mean-field theory based on the Bogoliubov - de Gennes (BdG) equations. We study the whole region along the BCS-BEC crossover including the unitary regime at zero temperature focusing on the situation in which the lattice potential is relatively weak as in the recent experiments. In such a situation, the tight binding description is not adequate and a full numerical approach based on the BdG equations is called for. Although approximate, this approach captures basic features along the whole BCS-BEC crossover, including the formation of molecules and the most challenging unitary limit where, for uniform 3D configurations, the predictions are in reasonably good agreement with ab initio Monte Carlo simulations. The BdG equations apply also to situations where the density varies over distances of the order of the healing length.

To study dynamical stability of period-doubled states, the following two methods can be taken: 1) linear stability analysis based on the framework of quasi-particle random phase approximation (QRPA) and 2) direct simulation of the time evolution by solving the time-dependent BdG equations.

Initially, we were going to take the first method. From the end of 2012, we had been working on programming to make a code for QRPA calculations. Although we spent more than one and a half years, we could not manage to make a QRPA code which gives correct results. Therefore, we changed the strategy and started to take the second method in 2014: studying the dynamical stability by directly following the time evolution based on the time-dependent BdG equations.

3. 結果

The change of the strategy was successful and we managed to study the dynamical stability based on the numerical simulations of the time-dependent BdG equations. We have found that the time scale for the onset of the dynamical instability of the period-doubled states drastically increases by going to the deeper BCS side so that the period-doubled states could be observed in current experiments. This is in a sharp contrast to the period-doubled states in the BEC regime whose timescale for the dynamical instability is very short.

4. まとめ

In summary, we have managed to perform the time-dependent BdG calculations to study the dynamical stability of the period-doubled states. We are now summarizing the outcome of our study and preparing the manuscript of the paper to be submitted.

We have found that period-doubled states exist as stationary states in Fermi superfluids and it can be energetically favorable compared to normal Bloch states in some parameter region. We have also found that the time scale for the onset of the dynamical instability of the period-doubled states drastically increases in the BCS regime.

5. 今後の計画・展望

To finalize the paper in preparation, we may need to perform some additional calculations. Together with Dr. Nakatsukasa, we are planning to start a new project to study the dynamical stability of the swallowtail in the superfluid Fermi gas along the BCS-BEC crossover. This is an important but unexplored problem; now, we can approach this problem with our machinery, which we have been developed for the last two years.

平成 26 年度　RICC 利用研究成果リスト

【論文、学会報告・雑誌などの論文発表】

Sukjin Yoon, Franco Dalfovo, Takashi Nakatsukasa, and Gentaro Watanabe:
"Multiple Period States of the Superfluid Fermi Gas in an Optical Lattice"
in preparation.

【国際会議、学会などでの口頭発表】

Gentaro Watanabe:
"Non-linear phenomena of superfluid Fermi gases in optical lattices"
Seminar of Department of Applied Physics in Aalto University
(Helsinki, Finland, 15 May 2013) [**invited**].

Gentaro Watanabe:
"Swallowtail band structure of the superfluid Fermi gas in an optical lattice"
Seminar at the Institute of Atomic and Molecular Sciences, Academia Sinica
(Taipei, Taiwan, 28 July 2014) [**invited**].