

**Project Title:**

**Numerical study of a universal four-component Fermi gas in one spatial dimension**

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**Background and Purpose:** Fermi gases have been the focus of intense study in recent years. The simplest example is a “unitary” Fermi gas: a gas of spin 1/2 Fermions with an attractive two-body interaction tuned to infinite scattering length and vanishing effective range. In this particular limit, there are no dimensionful scales to characterize the system other than its density. As such, dimensionful quantities such as the energy of the system must be proportional to appropriate powers of the density, and the proportionality constant is said to be universal. That is, it is completely independent of the details of the inter-particle potential in the universal regime. Theoretically, such a system is of great importance because it may be described by a conformal field theory. As such, many fascinating consequences may be derived, such as an operator-state correspondence that links the scaling dimensions of few-body operators in the theory to the energy levels of the few-body systems confined to a harmonic trapping potential. Unitary fermions also serve as an expansion point for an effective field theory description of nuclear physics. Experimentally, such a system may be realized using ultra-cold atoms tuned to a Feshbach resonance, thus allowing for a direct comparison with theoretical predictions.

Despite its simplicity, the spin  $\frac{1}{2}$  unitary Fermi gas is notoriously difficult to study theoretically due to the nonperturbative nature of the system. That is, because the scattering length is tuned to infinity, all fermions can feel the effects of all other fermions on all length scales, and therefore the system is strongly interacting. Due to an inherent lack of scales in the problem, no suitable dimensionless expansion parameters (such as ratios of two widely separated scales) exist to carry out a reliable perturbative analysis of the system. A nonperturbative description of the system exists in the context of a lattice effective field theory, although only in the simplest of situations—namely, when particle numbers and masses are equal, and when the interaction is attractive—can such a system be studied numerically. This is due to a “sign problem” arising from a complex action and resulting in the inapplicability of Monte Carlo techniques because of a non-positive probability measure for the path-integral representation for the partition function of the theory.

Recently, it was discovered that lower dimensional Fermi gases can exhibit many of the exact same properties as the three-dimensional Fermi gas described above. Particularly a four-component Fermi gas with an attractive four-body interaction tuned to the four-body resonance is described by a conformal field theory. As such, it exhibits many qualitative features in common with the three-dimensional theory, including universal few- and many-body physics, Effimov physics, and even an analogous operator-state correspondence. I recently demonstrated that a nonperturbative lattice definition of this model can be formulated in a way that is completely free of the sign problems that are prevalent in the three-dimensional theory. As a result, numerical study of this one-dimensional model could provide new qualitative and quantitative insights into the inaccessible regimes of such conformal theories.

**Usage status and calculational method:** The aim of this work is to numerically investigate a four-component Fermi gas in one dimension using a recently derived lattice construction, which is free of sign problems. Simulations were performed using lattice Monte Carlo methods with a local updating scheme based on the Metropolis method. Presently, I have performed few- and many-body numerical simulations for unitary fermions confined to both a harmonic trap, and a finite box. Few-body simulations of polarized and unpolarized systems were performed for up to eight fermions confined to a trap. Many-body simulations of unpolarized systems were performed for up to 56 fermions confined in a harmonic trap and 88 fermions confined to a finite box. In order to carry out continuum limit and infinite volume limit extrapolations of observables, each system of fixed fermion number was simulated at multiple lattice volumes. Estimates were performed on each ensemble to determine the kinetic, potential and interaction energies of the system. Currently all simulations were formed using approximately 350,000 core-hours on RICC and a comparable amount of resources obtained elsewhere.

**Results:** From estimates of energy observables, I have performed continuum limit extrapolations of such physically interesting quantities as the total energy (defined in three ways for trapped systems, using virial theorems) and integrated contact density of the system. For few-body trapped systems the energies measured provided an indirect

measurement of the scaling dimensions of few-body primary operators in free space. Continuum limit extrapolations of the energy and integrated contact densities for trapped and untrapped many-body systems allowed for two independent determinations of the Bertch parameter and its subleading correction due to a finite but large scattering length. A third determination of the Bertch parameter was achieved using estimates of the contact and additional theoretical input. All results were consistent with each other to within about one-percent statistical errors. Surprisingly, results for the Bertsch parameter for the unitary one-dimensional gas were also consistent to within 1% errors with the best experimental and theoretically determined estimates of the analogous Bertsch parameter for a three-dimensional two-component Fermi gas. This finding provides suggestive evidence for a deeper underlying relationship between two completely different universal Fermi gases. Since this parameter lies between zero and unity, the chances that the agreement is a coincidence is about 1/100 given current statistical and systematic uncertainties.

**Conclusion:** These studies represent a first step toward a complete quantitative understanding of a universal Fermi gas in one spatial dimension. Already this study has revealed an interesting possibility for a relationship between two conformal field theories in different space-time dimensions. The next natural step from a theoretical stand-point is to explore the reasons behind the connection. Should a rigorous explanation be found, this one-dimensional Fermi gas could provide a new and far more efficient way to numerically determine the three-dimensional Bertsch parameter to very high numerical precision.

**Schedule and prospects for the future:** In the future I wish to re-implement the simulations using an improved algorithm, such as continuous time Monte Carlo. Such improvements will hopefully improve the efficiency of the simulation by reducing auto-correlation times in the ensemble generation. With a new implementation, I would like to perform sub-percent level estimates of the same quantities described above. With theoretical justification for equivalence of Bertsch parameter in different dimensions, this would provide an estimate of the three-dimensional Bertsch parameter that is unmatched in conventional three-dimensional simulations. Also, there exists a different universal Fermi gas in one dimension that can be studied as a special limit of this theory. Numerical studies of the system could provide new insights into the nature of universal Fermi gases.

**Fiscal Year 2012 List of Publications Resulting from the Use of RICC**

**[Publications]**

“Transdimensional equivalence of universal constants for Fermi gases at unitarity”  
M. G. Endres, Phys. Rev. Lett. 109 (2012) 250403, [arXiv:1210.3104].

**[Oral presentations]**

“Universal four-component Fermi gas on the lattice”

The 29th International Symposium on Lattice Field Theory, Cairns, Australia, June 28, 2012

“Universal Fermi gases on the lattice”

National Chiao Tung University, Hsinchu, Taiwan, January 11, 2013.

“Universal Fermi gases on the lattice”

National Taiwan University, Taipei, Taiwan, January 9, 2013.

“Universal four-component Fermi gas on the lattice”

CASTS/LQCDHP Seminar, National Taiwan University, Taipei, Taiwan, May 25, 2012.