The looming crisis in air traffic capacity – what can vortex dynamics do?

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Dedicated to Kunio Kuwhara
Motivation:

- Circular jet
- Plane jet
- Mixing layer
- Cylinder wake
- Sheared vortex column
- Strained vortex column with CD

Core Dynamics (CD)
- Fl.Dyn.Res. '94

Generic CS
- Vortex column in turbulence
  - PRE '93
  - JFM '06

JFM '01
CS-turbulence interaction: Idealized flow

Idealizations:

• No interaction with other CS  
  no pairing or reconnection
• No background shear  
  no elliptic instability
• Rectilinear, cylindrical CS  
  no self-induced motion
• Random, fine-scale fluctuations  
  homog., isotrop. k – sep.

Flow evolution using DNS initialized with 3-D vort. from lin. analysis
Pseudo-spectral method (Rennich & Lele ’97; Pradeep & Hussain ’04)
periodic in z, pot. flow @ r → ∞

\[ \text{DNS} \quad \text{Re} \equiv \frac{\Gamma}{\nu} \quad 1k – 20k \]

Oseen-Lamb vortex
Numerical simulation method:

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Triply-periodic

```
V = 0
```

__“unbounded” flow__
(Rennich-Lele '97)
Pradeep & H. ‘04
q-vortex

$u_x$  $u_y$  $\omega$  

Box boundary

$V$  $W$

--- Theory

DNS
Comparison:

Unbounded BC

Periodic BC

time
Growth of q-vortex instability mode: Bending wave $\text{Re} = 10^5$ (DNS)
Vortex-Turbulence Interaction

turbulence

cohesive structure
dipoles

spiral threads
Transport effect of *threads*:
centrifugal instability

Self-limiting

Re = 5000
Mechanisms of core perturbation growth:

Centrifugal instability is self-limiting

Other mechanisms?

- Thread/Vortex wave resonance
- Transient growth
\( \gamma = 0.1 \)

\( \text{Re} = 2000 \)
TRANSIENT GROWTH
A rudimentary example:

TG: Temporary growth followed by decay
\[ V = \frac{\alpha}{r} + \frac{\beta r}{2} \]

model flow

\[ \beta/\alpha = \text{rotation/strain} \]

\[ \frac{\partial \omega_\theta}{\partial t} = \omega_r S \]

Higher strain S
i.e. \( \alpha \uparrow \)

\( \Rightarrow \omega_\theta \text{ gen.} \)

\( \Rightarrow \text{uv} \uparrow \)

\( \Rightarrow E \uparrow \)

inc. vort(\( \beta \))

\( \Rightarrow \text{arrest E} \uparrow \)

sooner

\[ E(t) \]

\( \beta/\alpha = 0 \)

\( \beta/\alpha = 0.05 \)

\( \beta/\alpha = 0.1 \)

\( \beta/\alpha = 0.2 \)

increasing vorticity

pure strain
LIN. INVISCID TG

Strain: unbounded growth (lin. sense)
   eventually saturate at NL level

core vorticity: arrest growth & period of growth
   → core oscillation

VISCOSITY damps both
Optimal gains: \textit{axi-sym} \hspace{1em} m = 0

\begin{figure}
\centering
\includegraphics[width=0.45\textwidth]{fig1a}
\includegraphics[width=0.45\textwidth]{fig1b}
\caption{G_{\text{max}} vs. k for different Re values.}
\end{figure}

Bending \hspace{1em} m = 1

\begin{figure}
\centering
\includegraphics[width=0.45\textwidth]{fig2a}
\includegraphics[width=0.45\textwidth]{fig2b}
\caption{G_{\text{max}} vs. k for different Re values.}
\end{figure}

Energy evolution:

\begin{figure}
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\includegraphics[width=0.45\textwidth]{fig3a}
\includegraphics[width=0.45\textwidth]{fig3b}
\caption{Energy E vs. r for different t values.}
\end{figure}
Re effect on tilting/stretching

\( m = 0 \)

\( m = 1 \)
TRANSIENT GROWTH

Nonlinear evolution of optimal modes
initial perturbation amplitude:

![Graph showing log(K) vs. time for different cases and initial amplitudes.]

**Case A**
- **Linear**
  - B: 0.6%
  - C: 2%
  - D: 6%
Structure at time of max. energy: \( m = 1 \) 

\[ \theta_\omega = 0.01 \]

\[ \omega = -0.05 \theta \]

\[ \omega = -0.3 \]

\[ \omega = 0.15 \theta \]

\[ \omega = 0.55 \theta \]

\[ \omega = 0.3 \theta \]

\[ \omega = 0.6 \theta \]

\( t = 90 \) \( t = 50 \) \( t = 50 \) \( t = 30 \)
Some conclusions

• Turbulence induces and amplifies core fluctuations – amplitudes exceeding those of external perturbations.

• Several potential mechanisms of core transition / accelerated vortex decay studied.

• Circulation overshoot $\Rightarrow$ centrifugal instability: amplifies perturbations, but inherently self-limiting.

• Weak “threads” can resonate with vortex core dynamics waves, but not strong perturbations.

• Transient growth: orders-of-magnitude amplification

• Strongest transient growth for bending waves.

Further study

*Nonlinear transient growth, regenerative transient growth, vortex breakup and turbulence self-sustenance*